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Probing Five-Dimensional Black Holes with D-Branes

Michael Douglas

Department of Physics and Astronomy, Rutgers University, Piscataway, NJ

Joseph Polchinski

Institute for Theoretical Physics, University of California, Santa Barbara, CA

and

Andrew Strominger

Department of Physics, University of California, Santa Barbara, CA

Abstract

We consider a one-brane probe in the presence of a five-dimensional black hole in the classical limit. The velocity-dependent force on a slowly-moving probe is characterized by a metric on the probe moduli space. This metric is computed for large black holes using low-energy supergravity, and for small black holes using D-brane gauge theory. The results are compared.

1. Introduction

Five-dimensional black holes in string theory [1] are primarily characterized by two quantities. The first is $gQ\alpha'$, where g is the closed-string coupling and Q is the number of minimal units of charge or constituent D-branes [2]. This is a measure of the size of the black hole. The second is g itself which governs the strength of quantum corrections.

In this paper we shall primarily consider the classical limit $g \to 0,\ Q \to \infty$ with the black hole size gQ held fixed in string units. For $g = 0$ there is no Hawking radiation. Within this limit there are two distinct regimes. Large black holes have $gQ \gg 1$ and are well described by general relativity or, more precisely, classical closed string perturbation theory. Small black holes have $gQ \ll 1$ and so are smaller than the size of a typical string. They are well described by D-brane perturbation theory [2]. The adjective "black" is in all cases appropriate for these objects[1] because, as pointed out in [3,4], the formula for their entropy implies that light cannot be emitted regardless of their size for $g \to 0$.

There is by now overwhelming evidence that for very low energy processes there is a single effective description – the effective string [1,5,6] – which is valid under some circumstances (the dilute gas regime [5,7]) for both large and small black holes. This effective description correctly yields the extremal [1,8] and near-extremal [5,9] entropies as well as the total Hawking radiation rate [10,11] and its functional dependence on various parameters [12,13,14]. Hence for low energy processes there is no qualitative difference between large and small black holes.

However a light, low energy probe misses a defining feature of a black hole: the event horizon. Such a probe is necessarily large and cannot fully fall through the event horizon. In order to directly see the event horizon of a small black hole, we need a probe that is smaller than the string length. In light of [15] D-branes are natural candidates and a series of works starting with [16,17] found that D-branes are quite effective at probing distances shorter than the string length. General arguments imply that this regime is well-described by D-brane world-volume gauge theory and numerous examples are given in [18].

Accordingly, in this paper we use D-branes to probe the structure of black holes. Specifically, we consider the moduli space metric for a wrapped one-brane probe in the presence of the five-dimensional black hole of [1]. Our results are puzzling and inconclusive.

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[1] One might question the use of the word "hole" for the small objects. However we shall later see that the probe moduli space metric has a hole in it.

For large black holes the moduli space metric is computed using a low energy spacetime-probe action. While the coefficients of the string metric are infinite series in $1/r$, we find that the probe moduli space metric has only $1/r^2$ and $1/r^4$ corrections. The $1/r^2$ piece follows simply from the long-range force laws. The $1/r^4$ term dominates the near-horizon behavior and has a universal, moduli-independent coefficient, reflecting the fact that near-horizon geometry forgets the asymptotic moduli[19]. Next we compare this to the D-brane calculation valid for small black holes. We expected – based on previous examples – that the D-brane calculation would produce the same metric. We indeed find that the $1/r^2$ term with the correct coefficient arises from a one-loop calculation in the D-brane gauge theory. However, although various considerations suggest that $1/r^4$ term should arise at two loops, we were unable to find it. Possibly we missed a subtlety in the two-loop calculation, as discussed at the end of Section 4. It is our view that resolution of this issue is crucial to further progress in this general direction.

This paper is organized as follows. Section 2 contains the basic results from the low energy classical closed string point of view – the derivation and justification of the probe action, and the demonstration that a probe approaching the black hole will be captured. We also consider general constraints on the probe metric, including those following from U-duality, and the behavior of a probe inside the event horizon. Section 3 extends this to near-extremal black holes, which exert a force on static probes. Section 4 analyzes the same system from the D-brane point of view. We derive the effective world-volume gauge theory on the probe and compute quantum corrections. Section 5 develops the constraints imposed by supersymmetery on the probe metric.

2. Slowly Moving One-branes and Five-branes in a Black Hole Background

2.1. The Extremal Black Hole Solution

The low-energy action for ten-dimensional type IIb string theory (in the notation of [7]) contains the terms,

$$\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{-g}e\left[e^{-2\phi}R+4e^{-2\phi}(\nabla\phi)^2-\frac{1}{12}H^2\right] \tag{2.1}$$

in the ten-dimensional string frame. H denotes the RR three form field strength, and ϕ is the dilaton. The NS three form, self-dual five form, and second scalar are set to zero.

We leave α' explicit in this section, but set it to one in Section 3. We let g denote ten-dimensional string coupling and define the zero mode of ϕ so that ϕ vanishes asymptotically. The ten-dimensional Newton's constant is then $G_{10} = 8\pi^6 g^2 \alpha'^4$. We wish to consider toroidal compactification to five dimensions with an S^1 of length $2\pi R$ and a T^4 of four-volume $(2\pi)^4 V$. With these conventions, T-duality sends R to α'/R or V to α'^4/V, and S-duality sends g to $1/g$.

A ten-dimensional extremal solution labeled by three charges is given by

$$e^{-2\phi} = \frac{Z_5}{Z_1}, \tag{2.2}$$

$$H = 2r_5^2 \epsilon_3 + 2r_1^2 e^{-2\phi} *_6 \epsilon_3, \tag{2.3}$$

$$\begin{aligned} ds^2 =& Z_1^{-1/2} Z_5^{-1/2} \left[-dt^2 + dx_5^2 + \frac{r_n^2}{r^2}(dt + dx_5)^2 \right] \\ &+ Z_1^{1/2} Z_5^{-1/2} dx^m dx^m + Z_1^{1/2} Z_5^{1/2} \left[dr^2 + r^2 d\Omega_3^2 \right], \end{aligned} \tag{2.4}$$

$$\begin{aligned} Z_1 &\equiv 1 + \frac{r_1^2}{r^2} \qquad r_1^2 \equiv \frac{gQ_1\alpha'^3}{V}, \\ Z_5 &\equiv 1 + \frac{r_5^2}{r^2} \qquad r_5^2 \equiv gQ_5\alpha', \\ Z_n &\equiv 1 + \frac{r_n^2}{r^2} \qquad r_n^2 \equiv \frac{g^2 n \alpha'^4}{R^2 V}, \end{aligned} \tag{2.5}$$

where $*_6$ is the Hodge dual in the six dimensions $x^0, .., x^5$ and ϵ_3 here is the volume element on the unit three-sphere. The event horizon is at $r = 0$. x^5 is periodically identified with period $2\pi R$, x^m, $m = 6, ..., 9$, are each identified with period $2\pi V^{1/4}$. The three charges are defined by

$$\begin{aligned} Q_1 &= \frac{V}{4\pi^2 g \alpha'^3} \int e^{2\phi} *_6 H, \\ Q_5 &= \frac{1}{4\pi^2 g \alpha'} \int H, \\ n &= RP, \end{aligned} \tag{2.6}$$

where P is the total momentum around the S^1. All charges are normalized to be integers and taken to be positive.

The entropy and energy are

$$\begin{aligned} E &= \frac{1}{g^2} \left[\frac{RgQ_1}{\alpha'} + \frac{RVgQ_5}{\alpha'^3} + \frac{g^2 n}{R} \right], \\ S &= 2\pi \sqrt{Q_1 Q_5 n} \ . \end{aligned} \tag{2.7}$$

2.2. The Classical Limit

In the classical limit the action becomes very large and the stationary phase approximation can be applied. Since the action (2.1) has an explicit $1/g^2$ prefactor, the limit $g \to 0$ with the fields held fixed is a classical limit. Noting the explicit factors of $1/g$ in the definitions (2.6) of the integer charges, as well as the explicit $1/g^2$ in the definition of the energy E and momentum P, this is equivalent to

$$g \to 0 \qquad \text{with} \;\; gQ_1, \;\; gQ_5, \;\; g^2 n \;\; \text{fixed.} \tag{2.8}$$

While the quantized charges diverge in the limit (2.8), the classical solutions remain finite. The canonical energy and momentum come with explicit $1/g^2$ factors, but this divergence is conventional and could be eliminated by using units in which $\hbar \neq 1$. However, the $1/g^2$ divergence of the entropy is meaningful.

Closed string perturbation theory naturally treats the fields ϕ, g and H as order one. Hence, noting the explicit factors of $1/g$ in (2.6), it is an expansion in g^2 with gQ_1, gQ_5 and $g^2 n$ fixed. Thus the classical limit (2.8) corresponds to genus zero closed string theory. A primary tool for analyzing black hole solutions in classical closed string theory is the α' expansion. The solutions (2.2)-(2.4) are solutions of the leading order α' equations. They are characterized by the squared length scales $gQ_1\alpha'$, $gQ_5\alpha'$ and $g^2 n\alpha'$ – in particular, the curvatures are bounded by these scales at the horizon. Thus, when these are large in string units:

$$gQ_1 > 1, \;\; gQ_5 > 1, \;\; g^2 n > 1, \tag{2.9}$$

the α' expansion is valid everywhere outside the horizon.

In section 4 we will compare this solution with the D-brane realization of the same black hole. D-brane perturbation theory involves both open and closed string loops. Closed string loops have factors of g^2, while open string loops have factors of gQ_1 or gQ_5, because the open string loops can end on any of the D-branes. Hence the classical limit (2.8) is a large N limit of the open string field theory. Closed string loops are suppressed, and the large N limit is the sum over planar open string diagrams with arbitrarily many boundaries.

This open string theory also admits an α' expansion, but now in powers of r^2/α', where r is a separation between D-branes, the parameter controlling the mass of the lightest open strings stretched between D-branes. When

$$r^2 \ll \alpha', \tag{2.10}$$

the qualitative physics and in particular the leading singular behavior of the theory is given by a world-volume quantum field theory keeping only these modes, while excited open string and closed string effects only make non-singular corrections [18]. Thus the near-horizon behavior should be described by the large N limit of a quantum field theory[2].

Note that (2.9) and (2.10) are conditions on different quantities and thus have a simultaneous regime of validity, the near-horizon behavior of a large black hole. Hence in principle large N D-brane gauge theory can be used to study the event horizon! Unfortunately, at present our ability to study the large N limit is limited. Most of what we know comes from taking the large N limit of exact results at finite N. The main result which is more general than this is large N factorization, which states that correlators of gauge invariant operators $\langle\prod_i \mathrm{Tr} O_i\rangle$ are dominated by the disconnected part $\prod_i\langle\mathrm{Tr} O_i\rangle(1+O(1/N))$. This translates into a precise sense in which the black hole is classical – if we express coefficients in the probe action in terms of gauge invariant operators formed from the black hole degrees of freedom, by evaluating the operators in the black hole configuration, we derive an action for probe motion in a constant background.*

For now, we will have to work with conventional open string perturbation theory. This is good if

$$gQ_1 < 1, \quad gQ_5 < 1, \quad g^2 n < 1 \tag{2.11}$$

and the regimes are mutually exclusive. We shall see that the last condition arises because a correlation function can pick up a factor of $g^2 n$ via propagators hooking on to the external state.

To summarize, the classical limit (2.8) may be characterized either by the classical genus zero closed string theory or by the large N limit of the quantum D-brane open string theory. These two different representations of the limit (2.8) truncate to field theory (and thus are useful) in different regimes of the couplings according to (2.9) and (2.11). The Lagrangian (2.1) is good for black holes large compared to the string scale, while the D-brane quantum field theory is good for small black holes.

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[2] Assuming, as we believe to be the case, that $r = 0$ in D-brane coordinates indeed coresponds to the horizon.

* One can also phrase this as the existence of a "master field" which determines gauge invariant quantities; see [20,21] for an introduction. A master field exists which reproduces any set of gauge invariant correlation functions.

2.3. One-brane and Five-brane probes

Consider a slowly moving D one-brane wound around the S^1 in the black hole background (2.2)- (2.4). It is convenient to choose static gauge $\tau = t, \quad \sigma = x^5$. The one-brane is then described by the ansatz

$$\begin{aligned} X^0 &= \tau, \\ X^5 &= -\sigma, \\ X^m &= X^m(\tau), \\ X^i &= X^i(\tau), \end{aligned} \tag{2.12}$$

where $i, j = 1, 2, 3, 4$ are indices in then non-compact transverse space and $m, n = 6, 7, 8, 9$ are internal T^4 indices. The one-brane worldsheet action, obtained for example by S-duality from the fundamental string action, is

$$-\frac{1}{2\pi g\alpha'}\int d\tau d\sigma\; e^{-\phi}\sqrt{-\det g_{\mu\nu}\partial X^\mu \partial X^\nu} + \frac{1}{2\pi g\alpha'}\int C^{(2)}, \tag{2.13}$$

where $C^{(2)}$ is the potential obeying $H = dC^{(2)}$and $\mu, \nu = 0, ...9$. The action governing the dynamics of a slowly moving one-brane in the gauge (2.12) is then obtained by inserting the ansatz (2.12) into this action. This yields

$$-\frac{R}{g\alpha'}\int d\tau + \frac{R}{2g\alpha'}\int d\tau\left(Z_n Z_5 v^2 + Z_n w^2\right) + \mathcal{O}(v^4), \tag{2.14}$$

where $v^2 \equiv \dot{X}^i\dot{X}^i$ is the squared transverse velocity and $w^2 \equiv \dot{X}^m\dot{X}^m$ is the squared velocity in the T^4. The first term in (2.14) represents the action of a one-brane at rest. In deriving this action we have used a cancellation (required by supersymmetry) between contributions from $g_{00}, \quad g_{55}$ and the two form potential C_{05} related to (2.3).

The motion of the one-brane is hence a geodesic in the wormhole geometry

$$ds^2_{M1} = Z_n Z_5\left[dr^2 + r^2 d\Omega_3^2\right] + Z_n dx^m dx^m. \tag{2.15}$$

Near $r = 0$ the first term approaches the flat metric on R^4

$$ds^2_{M1} \to \frac{g^3 n Q_5 {\alpha'}^3}{R^2 V}\left[d\rho^2 + \rho^2 d\Omega_3^2\right], \tag{2.16}$$

where $\rho = \alpha'/r$. Hence $r = 0$ is a second asymptotic region.

For the special case of $n = 0$, the metric (2.15) has appeared previously in the literature [22] as the transverse string metric of the symmetric five-brane carrying NS charge. This

is not a coincidence. Under S-duality Q_5 becomes the NS charge and the probe becomes a fundamental string. In this context it was argued that $(4,4)$ nonrenormalization theorems protect the metric from corrections. We expect the same to hold in the present context. For the special case of $Q_5 = 0$, (2.15) is S-dual to the metric for scattering of Dabholkar-Harvey winding states which appeared in [23,24]. These two special cases are related to one another by a U-duality transformation which exchanges n with Q_5.

Next let us consider the five-brane, with world-volume action proportional to

$$-\frac{1}{g\alpha'^3}\int d\tau d^5\sigma\ e^{-\phi}\sqrt{-\det g_{\mu\nu}\partial X^\mu \partial X^\nu} + \frac{1}{g}\int C^{(6)}. \tag{2.17}$$

Inserting an ansatz similar to (2.12) but with X^i wrapping the T^4 produces the same metric multiplied by the factor

$$e^{-2\phi}\mathrm{vol}\ ^2(T^4) = \frac{Z_1}{Z_5}. \tag{2.18}$$

Now the r-dependent terms in g_{00} and g_{ij} cancel against the six-form potential $C^{(6)}$ leaving the metric on the transverse moduli space

$$ds^2_{M5} = Z_n Z_1\left[dr^2 + r^2 d\Omega_3^2\right]. \tag{2.19}$$

Note that the transverse part of (2.15) and (2.19) are exchanged under T-duality of the internal T^4, as expected. One could also compute a metric for slow variations of the Wilson lines in the five-brane which would be T-dual to the second term in (2.15).

The simplicity of (2.15) and (2.19) is quite striking. While the black hole metric itself contains highly nonlinear corrections, these moduli space metrics truncate after $1/r^4$ corrections. This corresponds to at most one and two loops in the D-brane perturbation expansion.

2.4. Inside The Event Horizon

The black hole solution (2.4) has an event horizon at the coordinate singularity $r = 0$. The region inside the event horizon is described by the geometry

$$e^{-2\phi} = \frac{Z_5}{Z_1}, \tag{2.20}$$

$$H = 2r_5^2\epsilon_3 - 2r_1^2 e^{-2\phi} *_6 \epsilon_3, \tag{2.21}$$

$$
\begin{aligned}
ds^2 =& Z_1^{-1/2} Z_5^{-1/2} \left[dt^2 - dx_5^2 + \frac{r_n^2}{r^2} (dt + dx_5)^2 \right] \\
&+ Z_1^{1/2} Z_5^{-1/2} dx^m dx^m + Z_1^{1/2} Z_5^{1/2} \left[dr^2 + r^2 d\Omega_3^2 \right],
\end{aligned}
\tag{2.22}
$$

where in the interior region

$$
\begin{aligned}
Z_1 &\equiv -1 + \frac{r_1^2}{r^2}, \\
Z_5 &\equiv -1 + \frac{r_5^2}{r^2}.
\end{aligned}
\tag{2.23}
$$

This differs from the exterior geometry only in the sign of $dt^2 - dx_5^2$, signs in the definitions of Z_1, Z_5, and a sign in the second term in H. This last sign arises because dr is inward-pointing in the interior region (the horizon is still at $r = 0$). It has the interesting consequence [25] that only negatively charged, anti-one-branes are static inside the horizon.[3] If we further define

$$
Z_n \equiv -1 + \frac{r_n^2}{r^2}
\tag{2.24}
$$

in the interior region one finds that the anti-one-brane moduli space metric is given by (2.14) with the redefined Zs. Transforming to a new radial coordinate $\rho = \frac{r_n r_5}{r}$, the interior moduli space metric is

$$
ds_{M1}^2 = (1 - \frac{r_n^2}{\rho^2})(1 - \frac{r_5^2}{\rho^2})(d\rho^2 + \rho^2 d\Omega^2) + (\frac{\rho^2}{r_5^2} - 1) dx^m dx^m,
\tag{2.25}
$$

where now the horizon is at $\rho = \infty$.

2.5. U-Dual Moduli Space Metric

In this subsection we derive the manifestly U-dual expression which reproduces (2.14) and (2.19) for appropriate values of the charges. The bosonic terms of the low-energy effective action are manifestly U-dual in the five-dimensional Einstein frame [26]

$$
S = \frac{1}{4\pi^2 l_p^3} \left(\int d^5x \sqrt{-g} \left[R - \frac{\mathcal{G}_{\alpha\beta}}{2} \nabla_\mu \phi^\alpha \nabla^\mu \phi^\beta - \frac{\mathcal{M}_{IJ}(\phi)}{4} F_{\mu\nu}^I F^{J\mu\nu} \right] + ... \right),
\tag{2.26}
$$

where ϕ^α, $\alpha = 1, ..42$ parametrizes an $E_{6,6}/USp(8)$ coset with metric $\mathcal{G}$, upper (lower) $I = 1, ...27$ is an index in the 27 $(\bar{27})$ of E_6 and $F^I = dA^I$. An extremal black hole

[3] These have locally positive energy, but their energy as measured from spatial infinity in the exterior region is negative. Hence the black hole + probe configuration can still saturate the BPS bound. The no-force condition is satified for negative charges because the timelike black hole singularity has negative energy [25] (making it difficult to model with D-branes).

(probe) is characterized by 27 electric charges which we shall denote Q_I (q_I). The black hole (probe) mass M (m) and scalar charges Σ_α (σ_α) are determined by supersymmetry in terms of these 27 electric charges. The long range $1/r^3$ force between the black hole and probe is proportional to the products of the various charges. This force vanishes when the probe is stationary and if, as in the cases we consider, the black hole and probe charges preserve a common supersymmetry. This implies that the static interaction energy vanishes

$$\frac{l_p^3}{r^2}\left(\frac{mM}{3} - q_I(\mathcal{M}^{-1})^{IJ}Q_J + \sigma_\alpha(\mathcal{G}^{-1})^{\alpha\beta}\Sigma_\beta\right) = 0. \tag{2.27}$$

The quadratic velocity-dependent interaction energy can be deduced from the first quantized particle action

$$-\int d\tau\, \left\{(m + \sigma_\alpha\phi^\alpha)\det{}^{1/2}(g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu) + q_I A_\mu^I\dot{X}^\mu\right\}\ , \tag{2.28}$$

with α an arbitrary constant. Expanding to order v^2, the first, gravitational+scalar, term in (2.27) is multiplied by $1 - v^2$.[4] The electric term is uncorrected. Using the relation (2.27) between the charges and the moduli, the v^2/r^2 interaction energy can be written

$$\frac{l_p^3 v^2}{2r^2}\left(mM - q_I(\mathcal{M}^{-1})^{IJ}Q_J\right). \tag{2.29}$$

Next let us consider the $1/r^4$ terms in the moduli space metric. These control the $r \to 0$ limit, which is the near-horizon geometry. Thus they must be independent of the asymptotic moduli [19]. There is only one moduli-independent invariant that can be constructed from one probe and two black hole charges: $d^{IJK}q_IQ_JQ_K$ where d_{IJK} is proportional to the cubic E_6 invariant. The complete U-dual probe action is then

$$\int d\tau\, \left\{-m + \frac{v^2}{2}\left(m + \frac{mMl_p^3 - q_I(\mathcal{M}^{-1})^{IJ}Q_Jl_p^3}{r^2} + \frac{d^{IJK}q_IQ_JQ_Kl_p^3}{r^4}\right)\right\}. \tag{2.30}$$

For example, using the expression

$$l_p = \left(\frac{g^2\alpha'^4}{VR}\right)^{1/3} \tag{2.31}$$

for the five-dimensional Planck length in terms of the string conventions, the one-brane probe action (2.14) becomes

$$\int d\tau\, \left\{-\left[\frac{R^3}{l_p^3g^2V}\right]^{1/4} + \frac{v^2}{2}\left(\left[\frac{R^3}{l_p^3g^2V}\right]^{1/4} + \left(\frac{l_p^9}{g^2RV}\right)^{1/4}\frac{n}{r^2} + \frac{RQ_5}{r^2} + \frac{Q_5nl_p^3}{r^4}\right)\right\} \tag{2.32}$$

in the Einstein frame. As expected, the $1/r^4$ term does not depend on the moduli.

[4] The naive $v^2/2$ from the kinetic energy is only half the effect. The other half is from the spatial metric, and depends on the fact that $g_{ij} \sim (-g_{00})^{-1/2}\delta_{ij}$ for the five-dimensional black hole Einstein metric at long distance.

2.6. Geodesic motion

Moduli space is a euclidean space and so a moduli space metric cannot have an event horizon. Nevertheless we expect that the probe can fall behind the black hole event horizon and this should somehow show up in the moduli space metric. The existence of an event horizon shows up in the moduli space geodesics which describe the motion of the probe. While (2.15) and (2.19) are geodesically complete, there are some geodesics which fall into the wormhole and never return to the asymptotic region.[5] These correspond to probes which are captured by the black hole.

To analyze the geodesic motion consider an incoming one-brane with asymptotic velocity v (and $w=0$)and impact parameter b (in units of α'). Motion in (2.15) conserves the energy

$$E = \frac{p^2}{2Z_nZ_5} + \frac{L^2}{2r^2Z_nZ_5} \tag{2.33}$$

and angular momentum $L = r^2Z_nZ_5\dot{\theta}$. Here $p = Z_nZ_5\dot{r}$ is the canonical radial momentum. The asymptotic values are $E = v^2/2$ and $L = bv$. Solving for p produces

$$\begin{aligned} \frac{dr}{dt} &= \frac{1}{Z_nZ_5}\left(2EZ_nZ_5 - \frac{L^2}{r^2}\right)^{1/2} \\ &= \frac{v}{Z_nZ_5}\left(Z_nZ_5 - \frac{b^2}{r^2}\right)^{1/2} . \end{aligned} \tag{2.34}$$

The qualitative features of the motion can be understood by finding the turning points r_c at which $dr/dt = 0$. These are at

$$r_c^2 = \frac{b^2 - r_n^2 - r_5^2}{2} \pm \tfrac{1}{2}\sqrt{b^4 - 2b^2(r_n^2 + r_5^2) + (r_n^2 - r_5^2)^2}. \tag{2.35}$$

In the simplified case $r_n = 0$ (or $r_5 = 0$), there is a long tube but no second asymptotic region. For $b^2 > r_5^2$, the turning point is real, and the particle will re-emerge, after a time delay $\pi r_5^2/r_cv$. For $b \le r_5$, $\dot{r}$ never becomes zero, and the incoming probe will approach $r = 0$ monotonically. At late times $r \ll r_5$ and the solution behaves as $r \sim \exp -(2Er_5^2 - L^2)^{1/2}t/r_5^2$. Thus it is captured by the black hole.

[5] The fact that these geodesics take infinite time to reach the horizon at $r = 0$ reflects our use of Schwarzchild time in describing the motion.

More generally, the probe is captured if $b \leq b_c = r_n + r_5$. The wormhole geometry (2.15) has a minimal closed geodesic at $r_c = \sqrt{r_n r_5}$ which influences the motion. This can be seen by considering $b = b_c$. Now

$$\frac{dr}{dt} = \pm v \frac{1 - \frac{r_n r_5}{r^2}}{Z_n Z_5}. \tag{2.36}$$

At this critical impact parameter, the two real solutions of $\dot{r} = 0$ coalesce at r_c. Near r_c the motion is $\dot{r} \sim -vcr$ and the trajectory asymptotes to r_c. For $b < b_c$, the turning points move off the real axis. The probe is slowed down as it passes r_c, but will eventually asymptote to $r = 0$.

3. Static Probes of a Near-Extremal Black Hole

Supersymmetry implies that all configurations consisitng of a static one-brane probe plus an extremal black hole have degenerate energies. This degeneracy is lifted when supersymmetry is broken by excitng the probe. In the previous section the corresponding velocity and position dependent action was computed. In this section we shall consider the closely related problem in which the black hole rather than the probe is excited. The black hole then becomes near-extremal, the long range forces on a static probe no longer exactly cancel and the probe action acquires a potential term. We shall see that this potential has a structure similar to the metric of the preceding section.

3.1. Relevant Formulae for Near-Extremal Black Holes

The relevant formulae were collected in [7,12] from which most of this section was taken. In this section we set $\alpha' = 1$. We will work with the following near-extremal solution labeled by three charges [7], given in terms of the ten-dimensional variables by

$$e^{-2\phi} = \frac{Z_5}{Z_1}, \tag{3.1}$$

$$H = 2r_5^2 \epsilon_3 + 2r_1^2 e^{-2\phi} *_6 \epsilon_3, \tag{3.2}$$

$$\begin{aligned} ds^2 =& Z_1^{-1/2} Z_5^{-1/2} \left[-dt^2 + dx_5^2 + \frac{r_0^2}{r^2} (\cosh\sigma dt + \sinh\sigma dx_5)^2 \right] \\ &+ Z_1^{1/2} Z_5^{-1/2} dx^m dx^m + Z_1^{1/2} Z_5^{1/2} \left[Z_0^{-1} dr^2 + r^2 d\Omega_3^2 \right], \end{aligned} \tag{3.3}$$

where in this section we define

$$Z_1 = 1 + \frac{r_1^2}{r^2},$$
$$Z_5 = 1 + \frac{r_5^2}{r^2}, \qquad (3.4)$$
$$Z_0 = 1 - \frac{r_0^2}{r^2},$$

with

$$\begin{aligned} r_1^2 &= \sqrt{\left(\frac{gQ_1}{V}\right)^2 + \frac{r_0^4}{4}} - \frac{r_0^2}{2}, \\ r_5^2 &= \sqrt{(gQ_5)^2 + \frac{r_0^4}{4}} - \frac{r_0^2}{2}, \\ r_0^2 \frac{\sinh 2\sigma}{2} &= \frac{g^2 n}{R^2 V}, \\ r_n^2 &\equiv r_0^2 \sinh^2\sigma, \end{aligned} \qquad (3.5)$$

The extremal limit is $r_0 \to 0,\ \sigma \to \infty$ with n held fixed.

In the dilute gas region[6] defined by

$$r_0, r_n \ll r_1, r_5, \qquad (3.6)$$

the energy is approximately

$$E = \frac{1}{g^2}\left[RgQ_1 + RVgQ_5 + \frac{g^2 n}{R} + \frac{VRr_0^2 e^{-2\sigma}}{2}\right] . \qquad (3.7)$$

The momentum n is carried by a gas of left and right movers on the effective string. Equating the energy of this gas to $\frac{n}{R} + \frac{RVr_0^2 e^{-2\sigma}}{2g^2}$ and its momentum to $\frac{n}{R}$ we can determine n_L and n_R :

$$\begin{aligned} n_L &= n + \frac{R^2 V r_0^2 e^{-2\sigma}}{4g^2}, \\ n_R &= \frac{R^2 V r_0^2 e^{-2\sigma}}{4g^2}. \end{aligned} \qquad (3.8)$$

The left and right moving oscillations are governed by effective left and right moving temperatures

$$\begin{aligned} T_L &= \frac{1}{\pi}\frac{r_0 e^{\sigma}}{2r_1 r_5} = \frac{g}{\pi R r_1 r_5}\sqrt{\frac{n_L}{V}}, \\ T_R &= \frac{1}{\pi}\frac{r_0 e^{-\sigma}}{2r_1 r_5} = \frac{g}{\pi R r_1 r_5}\sqrt{\frac{n_R}{V}}. \end{aligned} \qquad (3.9)$$

[6] This corresponds to the limit $\alpha, \gamma \gg \sigma$ of the solution in [7], which is the dilute gas region discussed in [5][7][12].

Other useful relations are

$$r_0^2 = \frac{4g^2\sqrt{n_L n_R}}{R^2 V} = 4\pi^2 T_L T_R r_1^2 r_5^2, \tag{3.10}$$

$$e^{2\sigma} = \sqrt{\frac{n_L}{n_R}}. \tag{3.11}$$

3.2. The Probe Action

The action for a one-brane probe is

$$-\frac{1}{2\pi g}\int d\tau d\sigma\; e^{-\phi}\sqrt{-\det g_{\mu\nu}\partial X^\mu \partial X^\nu} + \frac{1}{2\pi g}\int C^{(2)}, \tag{3.12}$$

where

$$C^{(2)} = \frac{gQ_1 dt\wedge dx^5}{r^2 V Z_1} \tag{3.13}$$

is the potential obeying $H = dC^{(2)}$. For a static probe we take

$$\begin{aligned} X^0 &= \tau, \\ X^5 &= \sigma, \\ X^i &= constant, \\ X^m &= constant\ . \end{aligned} \tag{3.14}$$

For a static probe in an extremal black hole geometry there is an exact cancellation between the two contributions to the action. Hence there is no potential and no force on the probe. This cancellation no longer occurs when the black hole is excited. The first term is

$$-\int d\tau \frac{R\sqrt{Z_0}}{gZ_1} \tag{3.15}$$

while the second is

$$-\int d\tau \frac{RQ_1}{r^2 V Z_1}, \tag{3.16}$$

so the total action is

$$S = -\int d\tau \frac{R}{g}\frac{\sqrt{Z_0}+\frac{gQ_1}{r^2V}}{Z_1} \equiv -\int d\tau (\frac{R}{g} + U). \tag{3.17}$$

We wish to expand this in r_0. Note that

$$\sqrt{Z_0} = 1 - \frac{r_0^2}{2r^2} - \frac{r_0^4}{8r^4} + ... \tag{3.18}$$

$$Z_1 = 1 + \frac{gQ_1}{r^2V} - \frac{r_0^2}{2r^2} + \frac{r_0^4}{8r^2r_1^2} + ... \tag{3.19}$$

To leading order in r_0 the potential energy is then

$$U = -\frac{Rr_0^4}{8gr^2r_1^2} = -\frac{2g^2n_Ln_R}{R^3Vr^2Q_1} = -\frac{2\pi^4Rg^2Q_1Q_5^2T_L^2T_R^2}{r^2V}. \tag{3.20}$$

Let us now consider the case $T_L \gg T_R$ so that $n_L \sim n$ and $T_R \sim T_H/2$, where T_H is the Hawking temperature. Then

$$U \sim -\frac{\pi^2g^2nQ_5T_H^2}{2r^2RV}. \tag{3.21}$$

We have been working in string units. Transforming to five-dimensional Planck units using

$$\alpha' = (\frac{l_p^3VR}{g^2})^{1/4}, \tag{3.22}$$

the extra term in the action becomes

$$\Delta S = \int d\tau \frac{\pi^2nQ_5T_H^2l_p^3}{2r^2}. \tag{3.23}$$

Note that all moduli dependence has disappeared and the structure is similar to the $1/r^4$ term in the moduli space metric. It would be interesting to reproduce this term from D-brane perturbation theory.

4. Low Energy D-Brane Field Theory

We now consider the same system in the regime of D-brane perturbation theory. The black hole consists of Q_1 D one-branes parallel to the 5-direction and Q_5 D five-branes parallel to the 56789-directions, carrying 5-momentum P. The D one-brane probe is at a distance r small compared to the string scale. In this regime the relevant states are open strings that are massless or become massless as $r \to 0$. We will denote the string endpoints by 1, 5, and 1^*, the latter referring to endpoints on the probe.

The massless 1^*1^* states are a gauge field B^μ, a collective coordinate X^i (recall $i = 1, 2, 3, 4$), and their fermionic partners, together forming a vector multiplet, and a collective coordinate X^m ($m = 6, 7, 8, 9$) and its fermionic partners in a hypermultiplet. We are primarily concerned here with the effective action for X^i.

On the black hole are $Q_1^2 + Q_5^2 + Q_1Q_5$ hypermultiplets, of which $Q_1^2 + Q_5^2$ receive mass from the $U(Q_1)$ and $U(Q_5)$ D-terms [27]. It would thus appear to be natural to

represent the moduli as 15 open string fields. However, this description is simple only near the point in moduli space where these fields vanish. When these fields are large, the one-branes dissolve into the five-branes, becoming Q_1 $U(Q_5)$ instantons [28,29,30]. This instanton moduli space gives a global description of the black hole hypermultiplet moduli space. That is, on the five-branes is a self-dual gauge field $A_m(x^n, \zeta)$, where ζ are $4Q_1Q_5$ parameters. The low energy fields are given by letting ζ depend on x^μ, $\mu = 0, 5$. There are in addition $4Q_5^2$ scalars Y^i in the 55 vector multiplet. However, generically the instanton gas breaks the $U(Q_5)$ down to $U(1)$,[7] so only the center of mass part of Y^i is massless.

The interaction between the probe and the black hole comes from 1*5 strings with one end on each. Note that there are no 1*1 strings: we are in the regime where the one-branes are described by five-brane gauge fields, not by D one-branes. The 1*5 strings have mass proportional to r. Terms in the effective action which are singular in r are obtained by integrating out the virtual 1*5 strings. These fields comprise Q_5 hypermultiplets, with bosonic components $\phi^{A'a}$ and fermionic components χ^{Aa}. These are complex fields, with a a $U(Q_5)$ index. The indices A and A' are defined as follows. Call $SO(4)_E$ the rotational symmetry group of transverse space, and $SO(4)_I$ the local Lorentz symmetry in T^4. We index doublets of the two $SU(2)$'s in $SO(4)_E$ as A and Y, and doublets of the two $SU(2)$'s in $SO(4)_I$ as A' and $\tilde{A}'$.

To simplify the discussion we will take as external fields only the massless bosonic fields, namely B_μ, X^i, and X^n, the self-dual part of A_m, and the $U(1)$ parts of A_μ and Y^i. The full 1*5 hypermultiplets run in the loop. The minimally coupled action is

$$\begin{aligned} S_0 = &-\frac{\mu}{g}\int d^2x \biggl(1 + \frac{1}{2}\partial_\mu X^i \partial^\mu X^i + \frac{1}{4\mu^2} G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu X^m \partial^\mu X^m \\ &+ D_\mu \phi^\dagger D^\mu \phi + \mu^2 \left(X^{i2}\phi^\dagger\phi - 2X^i \phi^\dagger Y^i \phi + \phi^\dagger Y^{i2}\phi\right) \\ &+ \frac{\mu^2}{8}\left|\phi^{\dagger A'a}\phi_a^{B'} + \phi^{\dagger A'a}\phi_a^{B'}\right|^2 + \bar{\chi}(\Gamma^\mu D_\mu + i\mu\Gamma^i X^i)\chi\biggr) \\ &- \frac{\mu^3}{4\pi^2 g}\int d^6x \, \mathrm{Tr}\left(1 + \frac{1}{2}\partial_M Y^i \partial^M Y^i + \frac{1}{4\mu^2}F_{MN}F^{MN}\right) . \end{aligned} \tag{4.1}$$

Here μ is the fundamental string tension $(2\pi\alpha')^{-1}$; $G_{\mu\nu}$ is the 1*1* field strength and F_{MN} the 55 field strength, with M running over μ and m. The quartic potential terms can be understood repectively from the dimensional reduction of the $d = 6$ covariant derivative

[7] For a static configuration ($n = 0$), this is true only when $Q_1 \geq Q_5$. But when n is macroscopic it seems likely that the fluctuations fill out and break the full group.

and from the 1^*1^* and 55 $U(1)$ D-terms. The $SU(Q_5)$ D-terms must be absent (cancelled by exchange of massive fields) because this group is broken. There is also a five-brane $U(1)$ D-term which is omitted from (4.1) because it is proportional to $1/V$ and can be suppressed by large V.

The action for the black hole moduli ζ comes from the gauge kinetic term $F_{MN}F^{MN}$, specifically the components $F_{m\mu}F^{m\mu}$. In using the low energy effective action for the these gauge fields we are treating the instantons as large compared to the string scale. Effectively this is the first term in an expansion in $1/V$, since as the volume is scaled up the gauge fields scale uniformly. However, we will see that only one coupling matters, involving a conserved charge on the black hole side, and it is likely that its coefficient is universal.

In the minimal action (4.1) there is no coupling between the black hole moduli and the other fields. However, there are non-minimal (Born-Infeld-like) couplings that are important. Although nominally higher derivative, they contribute at leading order because of the nonzero 5-momentum of the black hole. To be somewhat systematic, note first that the fields and couplings appear in the D-brane action (4.1) and the expected moduli space metric (2.15) only in the combinations

$$\begin{array}{ccc} \alpha'^{-1}g:\ m^2 & \alpha'^{-1}X^i,\ \alpha'^{-1}X^m,\ B^\mu,\ A^\mu,\ \alpha'^{-1}\phi:\ m \\ \alpha'^{-1}\chi:\ m^{3/2} & \alpha' A_m:\ m^{-1} \qquad \alpha'^{-4}V:\ m^4 \end{array}$$

with units as shown. Any terms accompanied by explicit powers of α' in addition to these will be suppressed at low energy. This unusual dimensional analysis can be deduced by starting with the observation that the probe-black hole separation X^i enters only through the masses of the 1^*5 strings. It allows at the same order as the action (4.1) a number of new terms; the one relevant for the present purposes is $F_{-m}F_{-m}D_+\phi^\dagger D_+\phi$, through which the 1^*5 strings couple to the momentum carried by the black hole.

We can find this term and determine its coefficient by a T-duality argument, just as for the Born-Infeld action (see [31] for a review with references). Let us first find terms with $\partial_- X^m$. To do this consider a one-brane probe boosted so that X^m is linear in x^-,

$$X^m = u^m x^-. \tag{4.2}$$

In the rest frame of the probe the action is (4.1). The rest frame (primed) coordinates are related to the lab (unprimed) by

$$x'^- = x^-, \qquad x'^+ = x^+ - 2u_m x^m + u^2 x^-, \qquad x'^m = x^m - u^m x^-. \tag{4.3}$$

The spinor indices should be thought of as tangent space indices, so they do not transform. For 1^*5 fields, which live on $x'^m = 0$, the derivatives are related

$$\partial'_+ = \partial_+, \qquad \partial'_- = \partial_- + u^2\partial_+. \tag{4.4}$$

In the lab frame the action then has the additional terms

$$\delta_1 S = -\frac{\mu}{g}\int d^2x\, 2\partial_- X^m \partial_- X^m \left(\partial_+ X^i \partial_+ X^i + 2D_+\phi^\dagger D_+\phi + \bar\chi\Gamma_+ D_+\chi\right). \tag{4.5}$$

In addition, A_- picks up a non-gauge piece $A_m\partial_- X^m$. Terms involving F_{-m} are now obtained simply by T-duality. This takes $X_m \leftrightarrow \mu^{-1}A_m$ and gives

$$\begin{aligned}\delta_2 S = &-\frac{1}{g\mu}\int d^2x\, 2\mathrm{Tr}(F_{-m}F_{-m})\left(2D_+\phi^\dagger D_+\phi + \bar\chi\Gamma_+ D_+\chi\right)\\ &-\frac{\mu^3}{4\pi^2 g}\int d^6x\, \mathrm{Tr}\left(2F_{-m}F_{-m}\partial_+ Y^i\partial_+ Y^i\right).\end{aligned} \tag{4.6}$$

The modification of A_- is self-dual, up to charge conjugation and a gauge transformation.

The momentum carried by the black hole is

$$\frac{n}{R} = (2\pi)^5 RVT^{05} = \frac{(2\pi)^3 RV\mu}{g}\mathrm{Tr}(F_{-m}^2). \tag{4.7}$$

Hence u^2 maps to

$$\tilde u^2 = \frac{gn}{(2\pi)^3 R^2 V\mu^3 Q_5}, \tag{4.8}$$

with the assumption that F_{-m}^2 is on the average proportional to the identity. Collecting only the terms that survive in the low energy limit of interest gives the action

$$\begin{aligned}S = &-\frac{\mu}{g}\int d^2x\left(\frac{1}{2}\partial_\mu X^i\partial^\mu X^i + \frac{1}{4\mu^2}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu X^m\partial^\mu X^m\right.\\ &+\tilde D_\mu\phi^\dagger\tilde D^\mu\phi + \mu^2 X^{i2}\phi^\dagger\phi + \frac{\mu^2}{8}\left|\phi^{\dagger A'a}\phi_a^{B'} + \phi^{\dagger A'a}\phi_a^{B'}\right|^2\\ &\left.+\bar\chi(\Gamma^\mu D_\mu + i\mu\Gamma^i X^i)\chi + \frac{2}{\mu^2}\mathrm{Tr}(F_{-m}F_{-m})\left(2D_+\phi^\dagger D_+\phi + \bar\chi\Gamma_+ D_+\chi\right)\right)\\ &+\frac{\mu^3}{4\pi^2 g}\int d^6x\,\frac{1}{2\mu^2}\mathrm{Tr}\left(F_{-m}F_{+m}\right).\end{aligned} \tag{4.9}$$

Finally we can do the one loop graph 1^*5 graph. First go to the one-plus-one dimensional limit, $R\to\infty$, giving

$$\begin{aligned}&Q_5\log\det\left(\Gamma^\mu\partial'_\mu + i\mu\Gamma^i X^i\right) - 2Q_5\log\det\left(\partial'^\mu\partial'_\mu - \mu^2 X^{i2}\right)\\ =&\frac{Q_5}{2}\log\det\left(\partial'^\mu\partial'_\mu - \mu^2 X^{i2} + i\mu\Gamma^\mu\Gamma^i\partial'_\mu X^i\right) - 2Q_5\log\det\left(\partial'^\mu\partial'_\mu - \mu^2 X^2\right).\end{aligned} \tag{4.10}$$

Here we have defined

$$\partial'_+ = \partial_+, \qquad \partial'_- = \partial_- + \tilde{u}^2 \partial_+. \tag{4.11}$$

The only external 1^*1^* fields that have been kept are the X^i. The Dirac matrices are four-dimensional. The first nontrivial term is quadratic in the velocity, giving

$$-\mu^2 Q_5 \partial'^\mu X^i \partial'_\mu X^i \int \frac{d^2 p}{(2\pi)^2} (p'^\mu p'_\mu + \mu^2 X^2)^{-2} = -\frac{iQ_5}{4\pi}(1+\tilde{u}^2)\frac{\dot{X}^2}{X^2}. \tag{4.12}$$

The zero plus one loop effective action for the probe is then

$$-\frac{\mu}{2g}\dot{X}^2 \left(1 + \frac{Q_5 g}{\mu X^2} + \frac{g^2 n}{R^2 V \mu^4 X^2} \right) , \tag{4.13}$$

the same as the metric (2.15) from the supergravity regime, to this order.

For R finite one can make the usual long-string argument [6]. The effective length of the string is $2\pi R Q_1 Q_5$. In the classical limit this is long compared to Compton wavelength of the 1^*5 strings and so the above calculation still holds.

At two loops, where we might expect to see the full metric (2.15), there is a puzzle. There appear to be no two-loop graphs having r^{-4} singularities. All 55 fields are massive, as we have discussed, and give rise to low energy interactions that are higher order in the above dimensional analysis. Interactions involving 1^*1^* fields cannot contribute. If the 1^*1^* field is inside a loop the graph is suppressed by large-N counting in the classical limit. Otherwise, the graph must be one-particle-reducible with respect to the 1^*1^* field and so represents mixing (e.g. $X^i X^\mu$), but this is forbidden by symmetries. There remain only the quartic interactions among the 1^*5 fields (which, in superfield language, are actually 1PR on the 1^*1^* superfield). These give a figure-eight graph, with a 5-index running inside each loop and the 1^* around the outside, and with the external fields attached at various points. Now, both F_{-m} must attach to the same loop in order to give $\mathrm{Tr}(F_{-m})^2$. Lorentz invariance then requires both $\partial_+ X^i$ on that same loop, but then the other loop has no external attachments and vanishes by supersymmetry.

It is possible that the two loop term cannot be seen in the framework (4.1). We justified this expansion by considering a large-V limit where the generic gauge field becomes smooth. It may be that special points in the instanton moduli space, such as zero-size instantons, are the source of the missing term. This is under investigation.

5. Constraints from Supersymmetry

In this section we consider the constraints of supersymmetry on the moduli space metric. They are consistent with the results of the previous section.

First consider the limit $n = 0$, where the low energy theory has $(4,4)$ supersymmetry. The coordinates X^i are in an $(4,4)$ twisted chiral (=vector) multiplet [32]. This can be written in terms of two $(2,2)$ superfields, ϕ being chiral and χ twisted chiral. These satisfy

$$\begin{aligned} \bar{D}_+\phi &= \bar{D}_-\phi = 0 \\ \bar{D}_+\chi &= D_-\chi = 0. \end{aligned} \tag{5.1}$$

Their lowest components are

$$\phi|_{\theta=0} = X^3 + iX^4, \qquad \chi|_{\theta=0} = X^1 + iX^2. \tag{5.2}$$

The $N = 1$ action

$$S_1 = \int d^2x\, d\theta_+\, d\theta_-\, d\bar{\theta}_+\, d\bar{\theta}_-\, K(\phi, \bar{\phi}, \chi, \bar{\chi}) \tag{5.3}$$

gives the metric

$$ds^2 = K,_{\phi\bar{\phi}}\, d\phi d\bar{\phi} - K,_{\chi\bar{\chi}}\, d\chi d\bar{\chi}. \tag{5.4}$$

The condition for $(4,4)$ supersymmetry is [32]

$$K,_{\phi\bar{\phi}} + K,_{\chi\bar{\chi}} = 0, \tag{5.5}$$

which is simply Laplace's equation. The function K is not observable and need not be spherically symmetric, but Laplace's equation also follows for the metric compenents themselves and these must be spherically symmetric. Thus

$$g_{\phi\bar{\phi}} = g_{\chi\bar{\chi}} = a + \frac{b}{X^2}. \tag{5.6}$$

It follows that the dependence at $r \to \infty$ determines that at $r \to 0$. Similarly the $Q_5 = 0$ metric must be of the same form.

Now look at Q_5 and n both nonzero. The unbroken supersymmetry is $(4,0)$. A $(4,0)$ invariant operator with two lower $-$ indices must multiply n in the low energy theory.[8] This is easily written down with a $(4,0)$ superfield. Let us review the $(4,4)$ superfield for the twisted chiral multiplet [32]. It consists of two functions $\hat{\phi}$, $\hat{\chi}$, of $\theta_{a\pm}$ and their conjugates,

[8] It is conceivable that the full $(4,4)$ invariance will give a stronger constraint.

$a = 1, 2$. These satisfy differential constraints that completely determine their dependence on $\theta_{2\pm}, \bar{\theta}_{2\pm}$, so one can express everything in terms of the values at $\theta_{2\pm} = \bar{\theta}_{2\pm} = 0$; these are just the unhatted $(2, 2)$ superfields above, with $\theta_1 \to \theta$.

The same works for $(4, 0)$ with just the θ_{a+}. The action is

$$S_2 = n \int d^2x \, d\theta_{1+} \, d\bar{\theta}_{1+} \, d\theta_{2+} \, d\bar{\theta}_{2+} \, F(\hat{\phi}, \bar{\hat{\phi}}, \hat{\chi}, \bar{\hat{\chi}}) \tag{5.7}$$

and the constraints are [32]

$$\begin{aligned} &\bar{D}_{a+}\hat{\phi} = \bar{D}_{a+}\hat{\phi} = 0 \\ &(D_{2+} + i\bar{D}_{1+})(\bar{\phi} + \chi) = 0 \\ &(D_{2+} - i\bar{D}_{1+})(\phi + \bar{\chi}) = 0. \end{aligned} \tag{5.8}$$

Using these constraint one can reduce to $(2, 2)$ superfields,

$$\begin{aligned} \int d\theta_{2+} \, d\bar{\theta}_{2+} \, F \sim \; & D_{2+} \, \bar{D}_{2+} \, F\big|_{\theta_{2+}=\bar{\theta}_{2+}=0} \\ & = \left(F,_{\phi\bar{\phi}} + F,_{\chi\bar{\chi}}\right) \left(D_{1+}\phi\bar{D}_{1+}\bar{\phi} + D_{1+}\chi\bar{D}_{1+}\bar{\chi}\right)|_{\theta_{2+}=\bar{\theta}_{2+}=0} \; . \end{aligned} \tag{5.9}$$

The bosonic part of the action is then

$$\nabla^2 F n \partial_+ X^i \partial_+ X^i. \tag{5.10}$$

A general spherically symmetric F allows a general r-dependence.

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